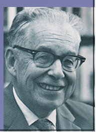
Lecture 3

Magnitudes & fractals

Local (Richter) magnitude

$$M_L=\log_{10}A-\log_{10}A_0(\Delta)$$

- M_L = local magnitude
- A = max amp in mm
- A_0 = max amp at distance Δ for standard (zero) earthquake (one which has an amplitude of 0.001 mm at 100 km)



Local (Richter) magnitude

$$M_L=\log_{10}A-\log_{10}A_0(\Delta)$$

In practice:

1. Measure amp
2. Calculate Δ
3. Look up $\log_{10}A_0(\Delta)$ in tables and subtract

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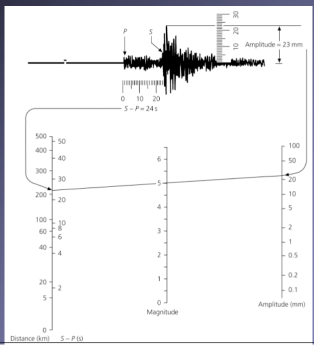
Calculation of local magnitude M_L



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Local (Richter) magnitude

$$M_L = \log_{10} A - \log_{10} A_0(\Delta)$$

Other important scales:

- Surface-wave magnitude M_S
- Body-wave magnitude m_b
- Duration magnitude M_d

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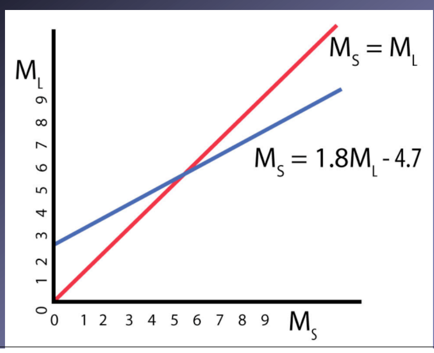
M_S *vs.* M_L



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Local (Richter) magnitude

$M_L = \log_{10}A - \log_{10}A_0(\Delta)$

Magnitude is not a fundamental property!

It is not meaningful to talk about the "accuracy" of a magnitude

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Empirical relationship with energy:

$\log E = 11.8 + 1.5M_S$

Question:

By what factor does the amount of energy released increase with each increase of one unity in M_S?

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Answer

$10^{1.5} = 31.6$

Intensity

Strength of ground shaking measured on Mercalli scale

I. People do not feel any Earth movement.
II. A few people might notice movement…
IV. Most people indoors feel movement…
VI. Everyone feels movement…
VIII. Drivers have trouble steering…
X. Most buildings & foundations destroyed…
XII. Objects thrown into the air…

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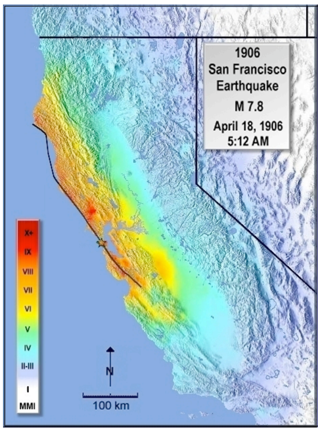

Intensity, 1906 San Francisco earthquake

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Seismic moment M_o

Earthquake size is given by the scalar seismic moment:

$$M_0 = \mu Au$$

μ = shear modulus

A = fault area

u = average slip (dislocation)

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1 Nm = 10^7 dyne cm

Seismic moment M_o

Calculation:

1. Using surface break & size of aftershock zone
2. Using spectral analysis

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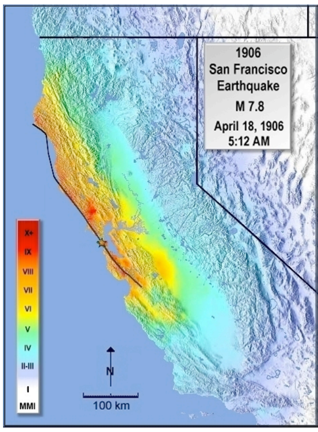

1906 San Francisco earthquake broke the surface

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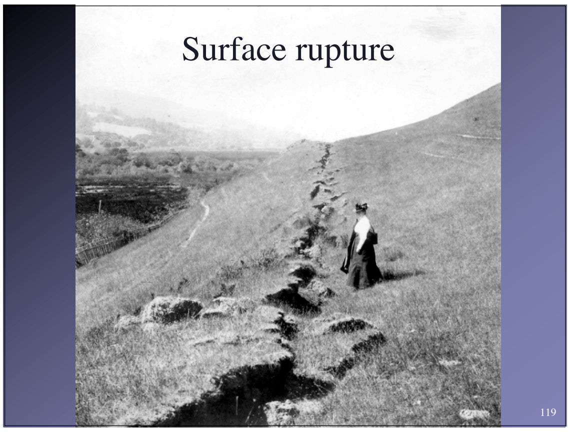
Surface rupture
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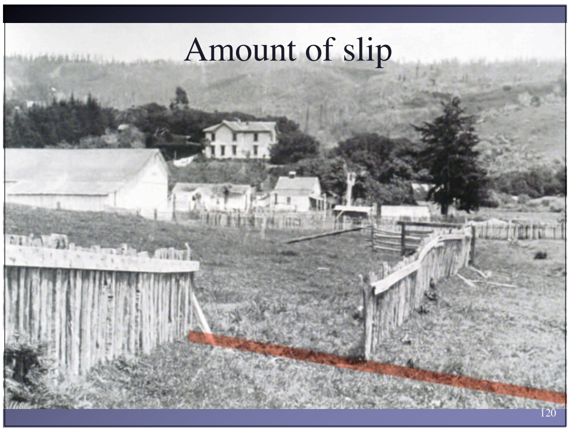
Amount of slip
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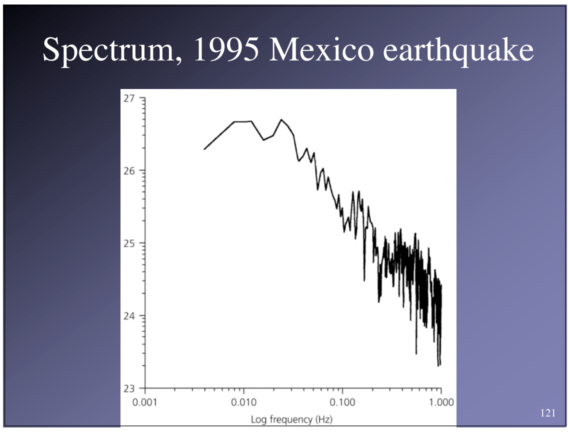
Spectrum, 1995 Mexico earthquake
27
26
25
24
23
0.001
0.010
0.100
1.000
Log frequency (Hz)
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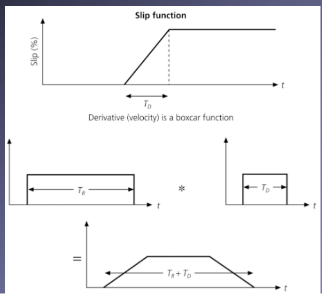
Trapezoidal source-time function



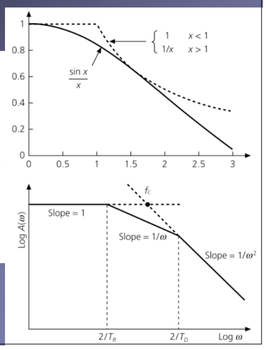
What would this source time function look like in the frequency domain?

Approximation of the amplitude spectrum

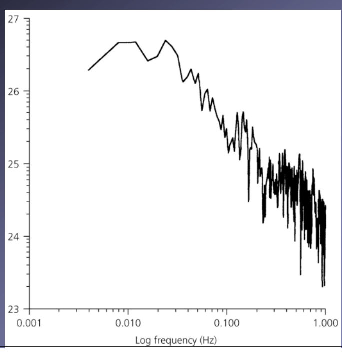
Approximation of the amplitude spectrum of a trapezoidal box car function:

$$\log|A(\omega)| = \left\{ \begin{array}{ll} \log M_0 & \omega < 2/T_R \\ \log M_0 - \log(T_R/2) - \log\omega & 2/T_R < \omega < 2/T_D \\ \log M_0 - \log(T_R T_D/4) - 2\log\omega & 2/T_D < \omega \end{array} \right\}$$

The spectrum os often defined in terms of the "corner frequencies"



Spectrum, 1995 Mexico earthquake



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$M_w = 2/3 \log M_o - 6.0$ (M_o in N m)

Empirical relationships with energy:

$$\log E = 11.8 + 1.5 M_S$$

$$M_0 = \mu A \bar{u}$$

$$\Delta\sigma = C\mu \frac{\bar{u}}{L}$$

$$E_S = \frac{1}{2}\Delta\sigma\bar{u}A$$

Thus:

$$E_S = \frac{\Delta\sigma}{2\mu} M_0$$

Moment magnitude (M_w), energy & stress

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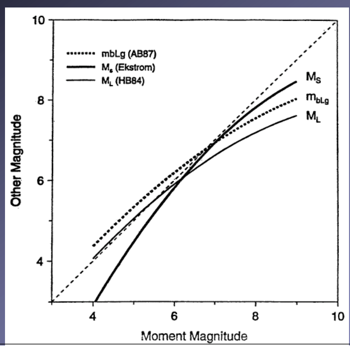
Moment & magnitude



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Stress & strain

Stress release in earthquakes is roughly constant:

$$\Delta\sigma = C\mu\ u/L$$

It is typically ~ 10 - 100 bars, which is much less than the strength of rock measured in laboratory experiments.

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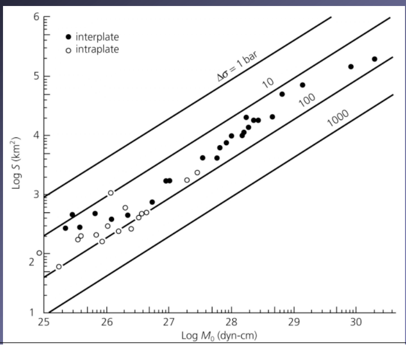
Empirical relations: rupture surface area *vs*. moment



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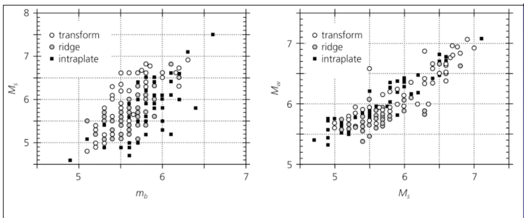
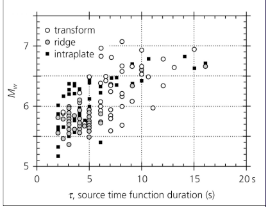
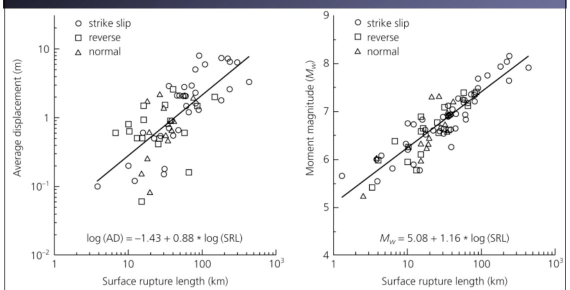




Empirical relations: M_o *vs*. magnitude & source duration

Empirical relations: slip & M_o *vs*. fault length



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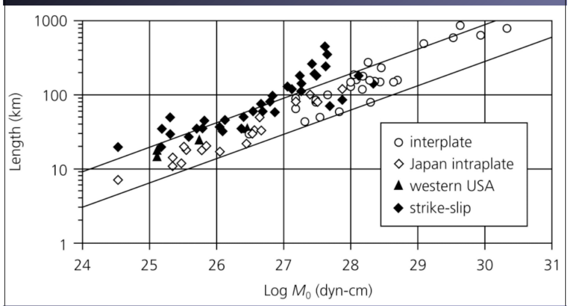
M_o *vs.* fault length



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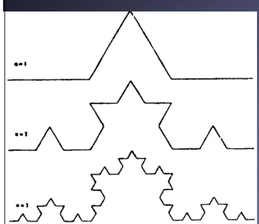
Fractals & *b*

Koch curve



$\mathbf{Nr^D = 1}$

D = fractal dimension

N = no. parts into which each line is split

r = ratio of similarity

Scale cannot be determined by examining a sample!

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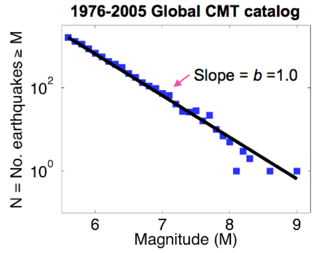
Magnitude relationships

The Gutenberg-Richter magnitude frequency relationship

1976-2005 Global CMT catalog



log(N) = a - bM

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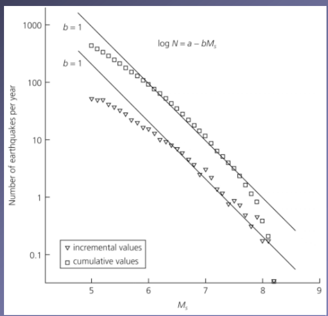
Frequency-magnitude plot for world, 1968-1997

Frequency - magnitude relation (fractal scaling; b-value)

$\log N = a_1 - bM$

N is the number of earthquakes with magnitude greater than M that occurred in a given time.

Because b is about 1, each year there is about 1 $M_s = 8$ earthquake, 10 $M_s = 7$ events, 100 $M_s = 6$ events, and so on.



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Maximum likelihood

$$\beta = \left[\bar{m} - \frac{(m_{MIN} - m_{MAX}\exp(-\beta(m_{MAX} - m_{MIN}))}{1 - \exp(-\beta(m_{MAX} - m_{MIN}))}\right]^{-1}$$

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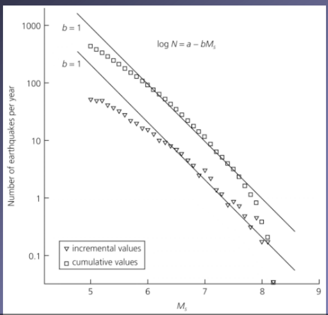
Deviations from linearity due to insufficient data



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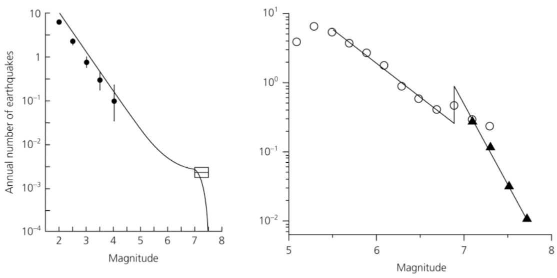
Deviations from linearity?



Wasatch fault zone, Utah

Continental Interiors

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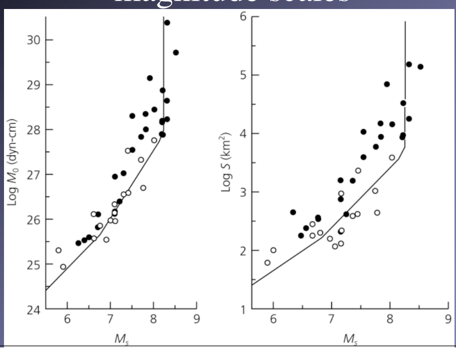
Saturation of body & surface wave magnitude scales



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Question

For smaller and larger earthquakes, consider the relative amounts of energy released and their relative numbers.

Is it possible that a large earthquake could be prevented in a seismogenic area by inducing many smaller earthquakes?

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Answer

No

Why not?

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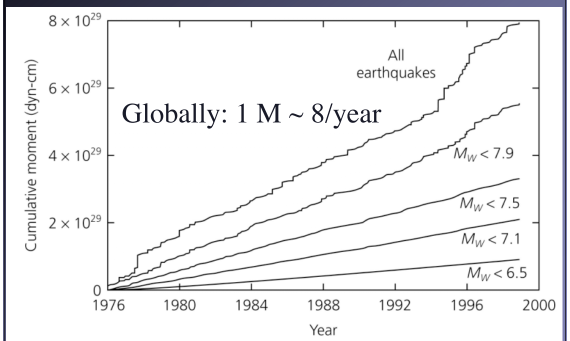
Cumulative seismic moment, world, 1976-1998



No. earthquakes/year

Earthquake Magnitude (M_s)	Number per year	Energy released (10^{15} J/yr)
≥ 8.0	0 - 1	0 - 1000
7 - 7.9	12	100
6 - 6.9	110	30
5 - 5.9	1400	5
4 - 4.9	13,500	1
3 - 3.9	> 100,000	0.2

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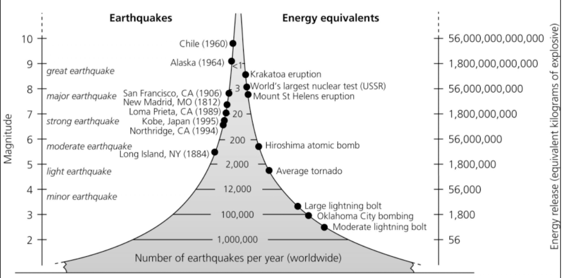
Frequency, magnitude & energy release



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Heart Attack	733,834
Cancer	544,278
Stroke	160,431
Lung disease	106,143
Pneumonia/Influenza	82,579
Diabetes	61,559
Motor vehicle accidents	43,300
AIDS	32,655
Suicide	30,862
Liver disease/Cirrhosis	25,135
Kidney disease	24,391
Alzheimer's	21,166
Homicide	20,738
Falling	14,100
Poison	10,400
Drowning	3,900
Fires	3,200
Suffocation	3,000
Bicycle accidents	695
Severe Weather[1]	514
In-line skating[2]	25
Football[2]	18
Skateboards[2]	10
Earthquakes (1811-1983)[3]	9
Earthquakes (1984-1998)	9

Causes of death in the US 1996

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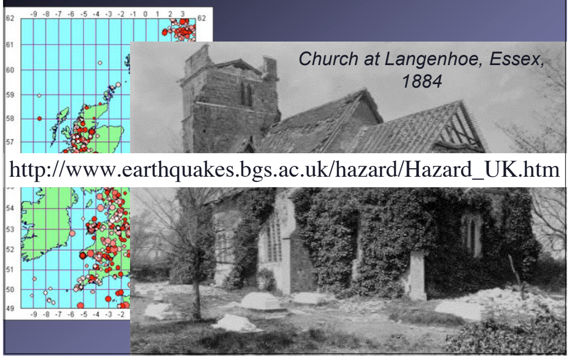
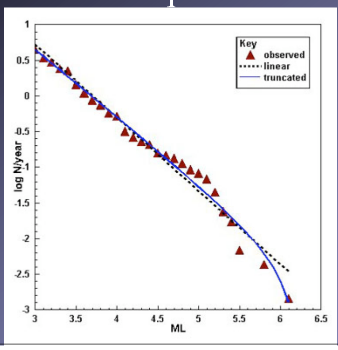
Earthquake hazard in the UK



http://www.earthquakes.bgs.ac.uk/hazard/Hazard_UK.htm

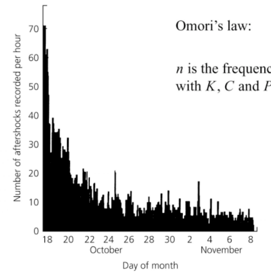
Frequency-moment plot for British earthquakes



Fall-off in frequency at high magnitude

1. Distribution asymptotic?
   - Can be processed using Gumbel statistics
   - Results:
     - max m_b = 5.7 for UK
     - max M_L = 9.2 for world
2. Distribution described by two different scalings with break point where whole crust ruptured

Aftershocks following 1989 Loma Prieta earthquake, Omori's law



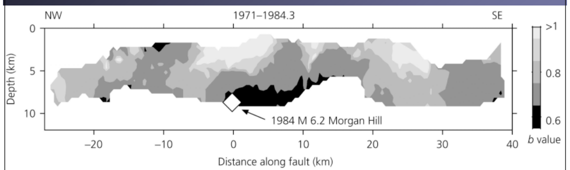
Omori's law: $n = \frac{C}{(K + t)^P}$

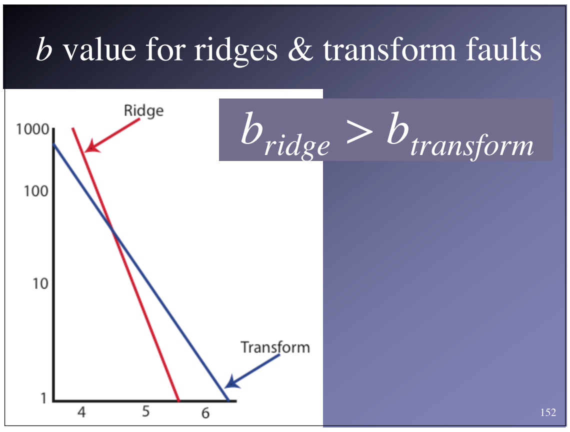
n is the frequency of aftershocks at a time t after the mainshock, with K, C and P as fault-dependent constants.

b varies in time & space

It is thought to be inversely proportional to stress

Variation in *b* along the Calavaras fault, California




b value for ridges & transform faults
$b_{ridge} > b_{transform}$
Ridge
Transform
1000
100
10
1
4
5
6
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